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Bayesian updating based model for the hydrodynamic added mass of the rectangular pile cap

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Abstract

The vibration of rectangular pile cap submerged in water can induce great hydrodynamic force that may exaggerate the structural seismic responses of bridges. This paper focuses on the calculation of the structural vibration-induced hydrodynamic added mass of the submerged rectangular pile cap. An efficient potential-based numerical method was firstly developed to determine the hydrodynamic added mass for immersed column with arbitrary cross-section. The general formulation of the hydrodynamic added mass was derived through the hydrodynamic force induced by the horizontal rigid motion of the column. The hydrodynamic added mass of an example column with circular cross-section was estimated through the developed model. The agreements with results from previous literatures indicated the accuracy of the developed model. To further validate the model through experiments, the vibration periods of an immersed cylinder under various water depths were obtained by both the developed model and experimental modal tests. Good agreements were found between the results from the developed model and experimental tests. Then, the developed model was adopted to determine the hydrodynamic added mass of 150 pile caps with different physical parameters sampled by Latin hypercube sampling. The calculated added mass and associated physical parameters were collected to build a database. A Bayesian updating based model for the hydrodynamic added mass of the submerged rectangular pile cap was proposed. The built database was used to update the proposed model and evaluate the posterior statistics of the unknown model parameters. Finally, the equation for the mean value of hydrodynamic added mass was expressed as a function of several physical parameters of the submerged rectangular pile cap for engineering application. In order to assess the precision of the proposed model, experimental modal tests of an isolated rectangular pile cap specimen with different submerged depths were carried out. The numerical model of the pile cap specimen was also established using the hydrodynamic added mass calculated by the proposed model. Numerical results of modal periods of the pile cap were obtained and compared with the experimental ones. The consistency between the periods determined using the proposed model and the experimental results of the tested pile cap revealed that the proposed Bayesian updating based model can accurately predict the hydrodynamic added mass of submerged rectangular pile cap.

Keywords: Bayesian updating; hydrodynamic; added mass; immersed column; pile cap; modal test

1. Introduction

In recent decades, pile foundations have been widely used to support long-span deepwater bridges. The pile cap in the pile foundations of these bridges, typically a large rectangular concrete block, is always partially or fully submerged in water. Because of its large dimension, the pile cap is exposed to large hydrodynamic force, which may significantly increase the structural seismic responses [1]. Therefore, it is important to evaluate the hydrodynamic effect on the submerged pile cap. In engineering application, the hydrodynamic effects are usually transferred to hydrodynamic added mass. To this end, many equations have been developed to calculate the hydrodynamic added mass for immersed columns, which are expressed as functions of the cross-section dimensions and water depth [2]. Therefore, it is desired to propose an added mass equation for the submerged rectangular pile cap as a function of its geometrical parameters. Inspired by the Bayesian probabilistic models [3], this paper proposed a Bayesian updating based model for the hydrodynamic added mass of the submerged rectangular pile cap.

2. Organization

2.1 Model for hydrodynamic added mass of immersed column with arbitrary cross-section

An immersed column with arbitrary cross-section subjected to horizontal ground acceleration is studied, as illustrated in Fig. 1. The general form of hydrodynamic added mass per unit height at z_j of this column vibrating along the direction θ_g is denoted as $m_a(z_j, \theta_g)$, which can calculated by

$$m_a(z_j, \theta_g) = \int_{S(r,\theta,z_j)} p(r,\theta,z_j)\vec{\mathbf{n}}_s \cdot \vec{\mathbf{n}}_g \mathrm{d}S \quad (1)$$

where $p(r,\theta,z_j)$ is the hydrodynamic pressure acting on the column surface induced by its rigid motion; $\vec{\mathbf{n}}_s$ is the unit normal vector of surface function $S(r,\theta,z_j)$ pointing outwards, $\vec{\mathbf{n}}_g$ is the unit directional vector of ground motion along the direction θ_g, and $\cdot$ is the dot product between vectors.

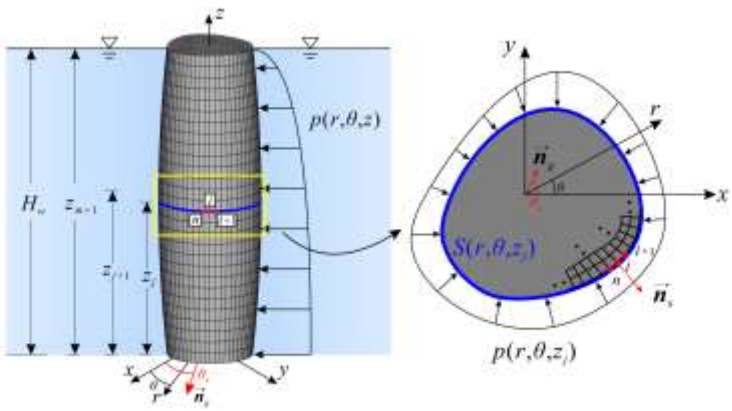

Figure 1 Illustration of an immersed column with arbitrary cross-section subjected to horizontal ground motion.

In order to compute the hydrodynamic pressure on columns with arbitrary cross-section, the column and the surrounding water are discretized into solid elements and potential-based fluid elements, respectively. Then, an analysis process can be performed to calculate the hydrodynamic added mass of the immersed column with arbitrary cross-section [4].

To assess the accuracy of the developed model, the hydrodynamic added mass of an immersed column with circular cross-section, as shown in Fig. 2, is calculated by the developed model and the results are compared with those determined using the analytical equations from previous literatures [5]. Morever, experimental modal tests of an immersed cylinder are conducted in a water tank, as shown in Fig. 3. The period ratios of the immersed cylinder under different water depths are obtained experimentally and numerically using the added mass determined by the developed model to further verify the model.

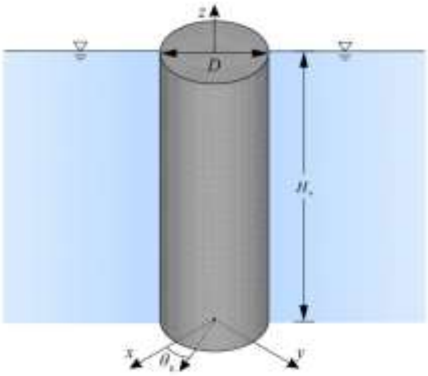

Figure 2 Dimensions of immersed column with circular cross-section (D = 4 m, H_w = 30 m).

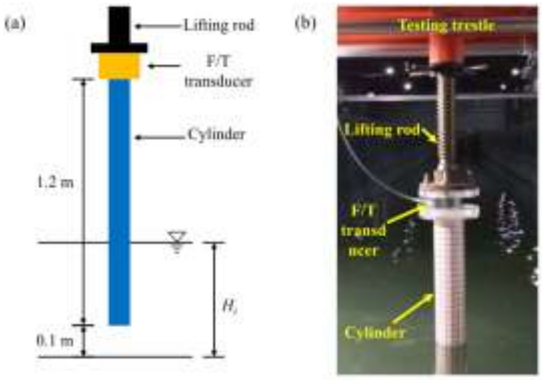

Figure 3 Experimental setup: (a) positions of the cylinder and F/T transducer; (b) global view of the cylinder and the testing setup.

2.2 Bayesian updating model for added mass of pile cap

As illustrated in Fig. 4, a submerged rectangular pile cap vibrating horizontally with acceleration $\ddot{u}_g$ can be characterized by four physical parameters: D and L are the length of cap side normal and parallel to the vibration direction, respectively; t is the cap thickness; and h is the submerged depth, which is the distance between the cap bottom and water surface.

The expression of the proposed Bayesian updating model can be written as follows,

$$M(\mathbf{x},\mathbf{\Theta}) = \hat{m}(\mathbf{x}) + \gamma(\mathbf{x},\mathbf{\theta}) + \sigma\varepsilon \tag{2}$$

where $M(\mathbf{x},\mathbf{\Theta})$ is the hydrodynamic added mass of the submerged rectangular pile cap; $\mathbf{x}$ is the physical parameters of pile cap; $\mathbf{\Theta} = (\mathbf{\theta},\sigma)$ is a set of unknown model parameters, and σ is the unknown standard deviation of model error; $\hat{m}(\mathbf{x})$ is a deterministic model; $\gamma(\mathbf{x},\mathbf{\theta})$ is the correction term; and ε is a random variable following standard normal distribution.

Firstly, the Latin hypercube sampling is used to generate a total of 150 pile cap samples. The added mass for each pile cap sample is computed using the developed model to create a database. Secondly, the Bayesian updating theory and the database are used to estimate and update the unknown model parameters. Finally, the equation for the mean value of hydrodynamic added mass of the submerged rectangular pile cap is written as follows:

$$\begin{aligned} & m_a^{mean} = \rho_w DLt \cdot 0.5742(\frac{D}{L})^{\beta_1}(\frac{D}{s})^{\beta_2}(\frac{L}{t})^{\beta_3}(\frac{s}{h})^{\beta_4}(\frac{D}{h})^{\beta_5}(\frac{h}{t})^{\beta_6} \\ & D/L \in [0.4, 2.5], D/t \in [2,10], h/t \in [0.4, 3.5] \end{aligned} \tag{3}$$

where s is the submerged height of pile cap, it equals t when h is larger than t and equals h otherwise; and $\beta_i (i = 1, 2, ..., 6)$ is expressed as:

$$\begin{aligned} \beta_1 &= 1.8812 \\ \beta_2 &= -1.6299 \\ \beta_3 &= 0.9101 \\ \beta_4 &= -0.5111 \\ \beta_5 &= -0.077\ln(D/h) \\ \beta_6 &= -0.2727\ln(h/t) \end{aligned}$$

In order to examine the precision of the proposed Bayesian updating model, experimental modal testings for a submerged rectangular pile cap are carried out, as shown in Fig. 5. A numerical model of the pile cap with added mass calculated using the propsed model is also built.

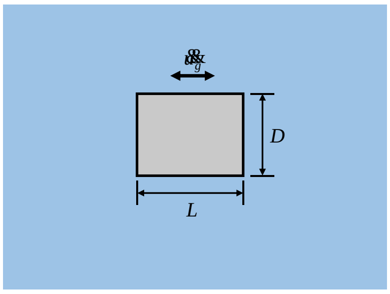

Figure 4 Illustration of a rectangular pile cap vibrating in water.

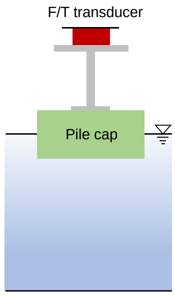

Figure 5 The tested pile cap and F/T transducer (D=0.384 m, L=0.384 m, t=0.16 m).

3. Results and Discussions

Fig. 6 shows the comparison of hydrodynamic added mass coefficient between the developed model and the analytical equations. It is clear that the results predicted from the developed model agree well with the closed-form results from literatures for the circular column. Fig. 7 compares the measured and computed wet-to-dry period ratios. It can be seen that the numerical period ratios coincide with the experimental ones, which demonstrates the high accuracy of the developed model for added mass evaluation.

Tab. 1 compares the experimental results of the period with the mean values of period calculated according to the Bayesian updating model for the tested pile cap. The periods from the Bayesian updating model agree well with the experimental results. Only small discrepancies can be observed. It can be concluded that the proposed Bayesian updating model has high accuracy to assess the hydrodynamic added mass of the submerged rectangular pile cap.

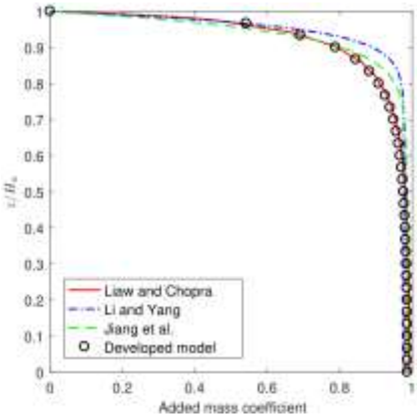

Figure 6 Comparison of added mass coefficient between results from developed model and literatures.

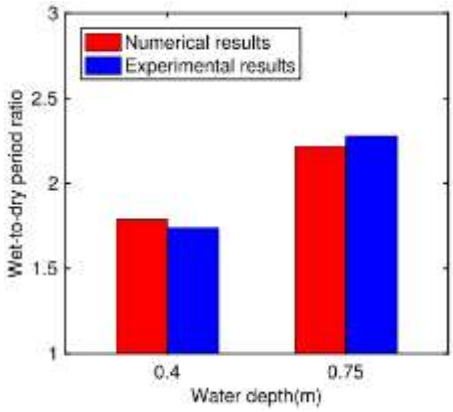

Figure 7 Numerical and experimental results of wet-to-dry period ratios for the cylinder as a function of water depth.

Table 1 Comparison of periods from the Bayesian updating model and experimental results.

Water depth/m	Bayesian updating model/s	Experimental results/s	Error(%)
0.08	0.090	0.090	0.0
0.16	0.106	0.107	0.9

4. Acknowledgements

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5. References

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