Bayesian Updating Based Model for the Hydrodynamic Added Mass of the Rectangular Pile Cap

Kai Wei

PhD., Associate Professor

Department of Bridge Engineering

Southwest Jiaotong University

Outline

1. Motivations
2. Potential-based added mass calculation
3. Added mass model of the cap
4. Concluding Remarks

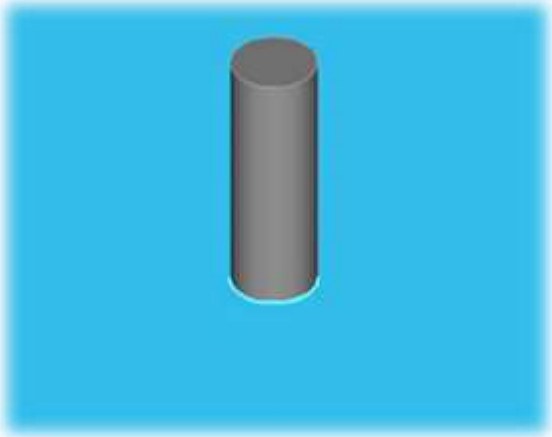
Motivations

- <u>Seismic hazards of deepwater bridges</u>

Deepwater bridges

Earthquake

Different from the bridges on land



Fluid-structure interaction

Beam dropped down

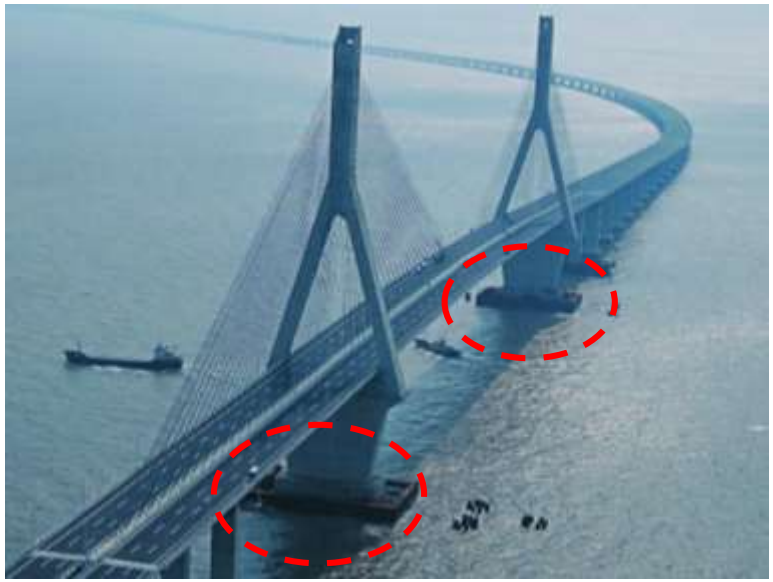
Miaoziping Bridge

Two cracks at main pier

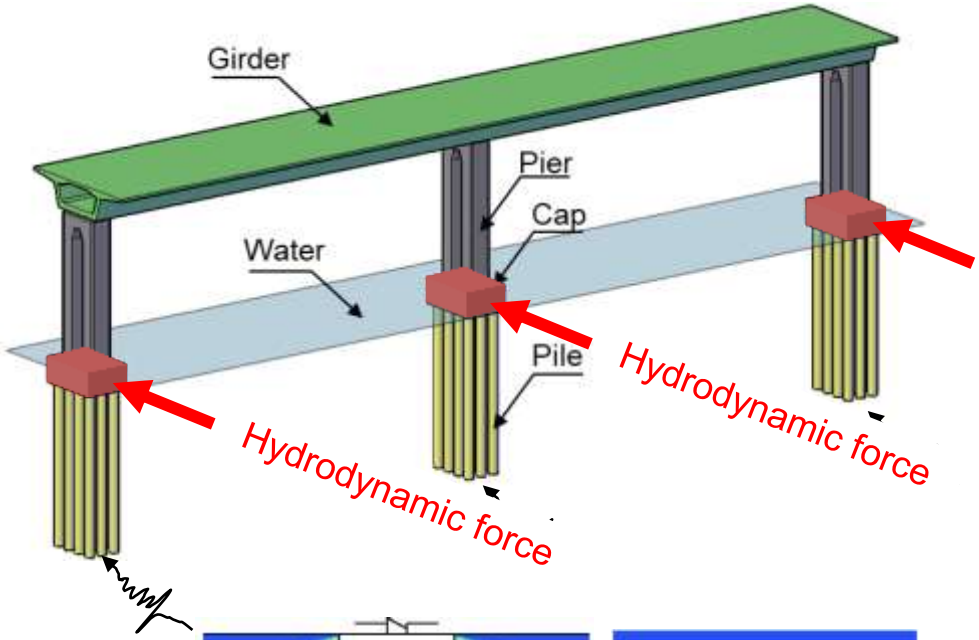
Rehabilitation cost: 1 year and $ 4,500,000!

Motivations

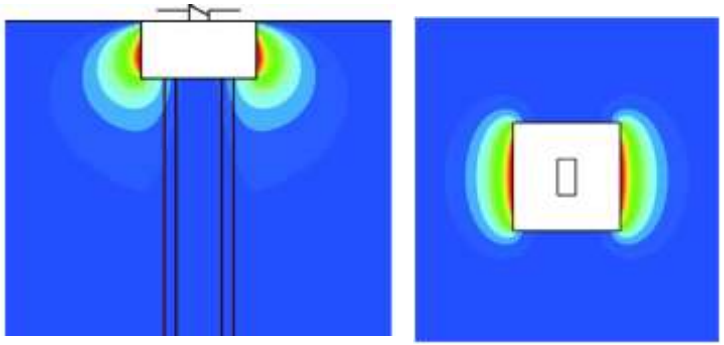
- <u>Special hydrodynamic problem for bridge pile group</u>



East Sea Bridge



- Pile cap foundations are widely used in long-span deepwater bridges
- Hydrodynamic force on the cap dominates structural seismic response



Significant hydrodynamic pressure around pile cap

Motivations

- Consideration of hydrodynamic effect in design codes

Eurocode 8

$$m_w = \rho_w \pi \frac{D^2}{4}$$

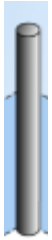
Japanese Code

$$m_w(y) = \begin{cases} \frac{b}{a}\left(1 - \frac{b}{4h_w}\right)\sqrt[3]{\frac{y}{h_w}}\rho_w A & b/h_w \le 2 \\ \frac{b}{a}\left(0.7 - \frac{b}{10h_w}\right)\sqrt[3]{\frac{y}{h_w}}\rho_w A & 2 < b/h_w \le 4 \\ \frac{3b}{10a}\sqrt[3]{\frac{y}{h_w}}\rho_w A & 4 < b/h_w \end{cases}$$

Chinese Railway Code

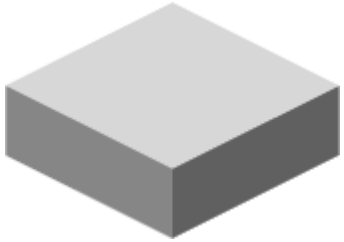
$$m_w(z) = \begin{cases} \rho_w A & 0 < z \le 0.8h_w \\ \frac{5(h_w - z)}{h_w}\rho_w A & 0.8h_w < z \le h_w \end{cases}$$

Pier
Pile



D/h < 0.2

While pile cap is not in the scope.



- Analytical method for cap

Non-circle sections: closed-form solution is not available.

<u>Objective</u>: proposing a general method to calculated added mass on the cap.

Potential-based added mass calculation

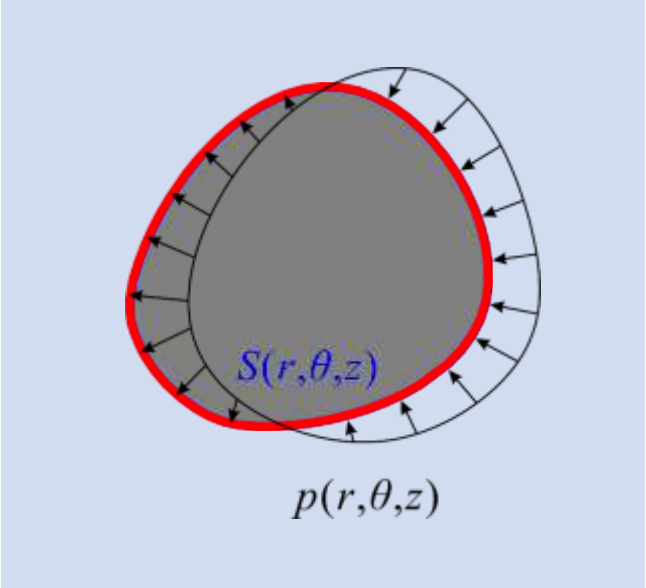
- How to calculate added mass

"Added mass can be calculated according to the hydrodynamic force on the outer surface of the vibration structure under rigid body assumption."

Added mass = Hydrodynamic force / $\ddot{u}_g$

——A. K. Chopra

If $\ddot{u}_g = 1$, Added mass = Hydrodynamic force ?



? Hydrodynamic pressure $p(r,\theta,z)$

Integration

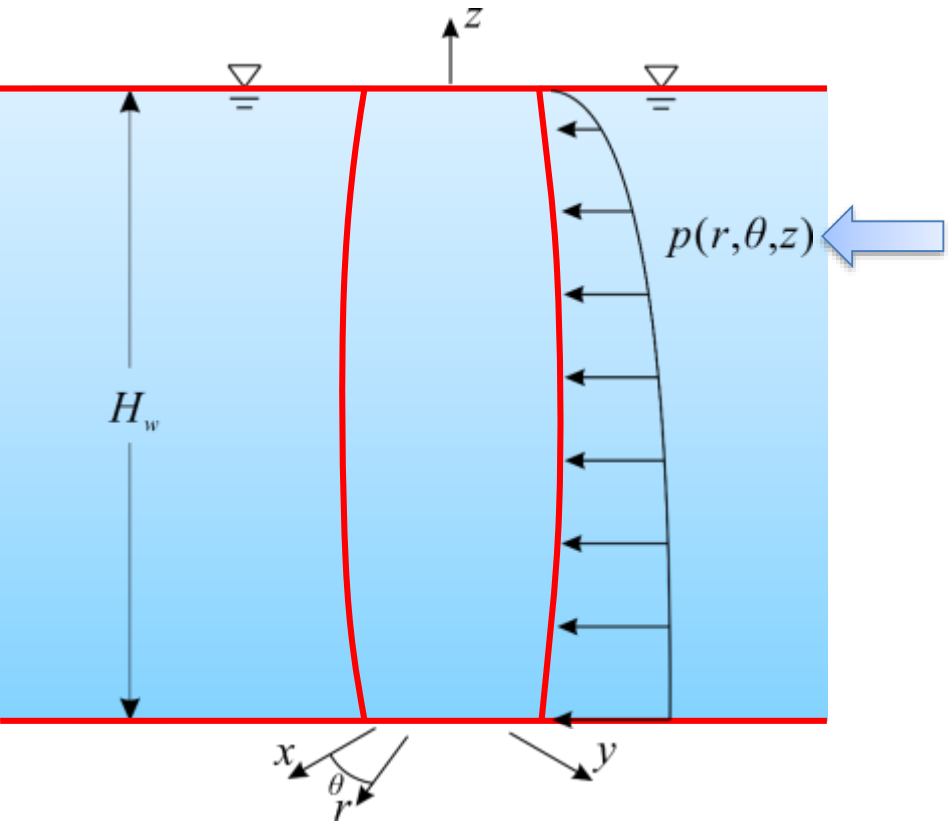
Hydrodynamic force $F_p(z) = \int_{S(r,\theta,z)} p(r,\theta,z)\vec{\mathbf{n}}_s \cdot \vec{\mathbf{n}}_g \mathrm{d}S$

Added mass

Potential-based added mass calculation

- How to calculate hydrodynamic pressure

For complex column, hydrodynamic pressure can be computed using **potential-based finite element method**



A submerged column moves rigidly with unit acceleration

Governing equation in water domain

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{C^2}\frac{\partial^2 p}{\partial t^2}$$

Boundary condition 1: Free surface

$$p(r, \theta, H_w, t) = 0$$

Boundary condition 2: Water bottom

$$\frac{\partial p}{\partial z}(r, \theta, 0, t) = 0$$

Boundary condition 3: FSI interface

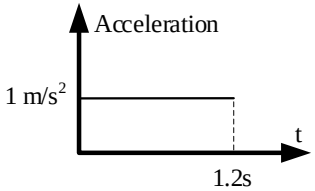
$$\frac{\partial p}{\partial \vec{\mathbf{n}}_s}(r, \theta, z, t)|_{S(r,\theta,z)} = -\rho_w \ddot{u}_g(t)\, \vec{\mathbf{n}}_s \cdot \vec{\mathbf{n}}_g$$

Potential-based added mass calculation

- Numerical analysis framework

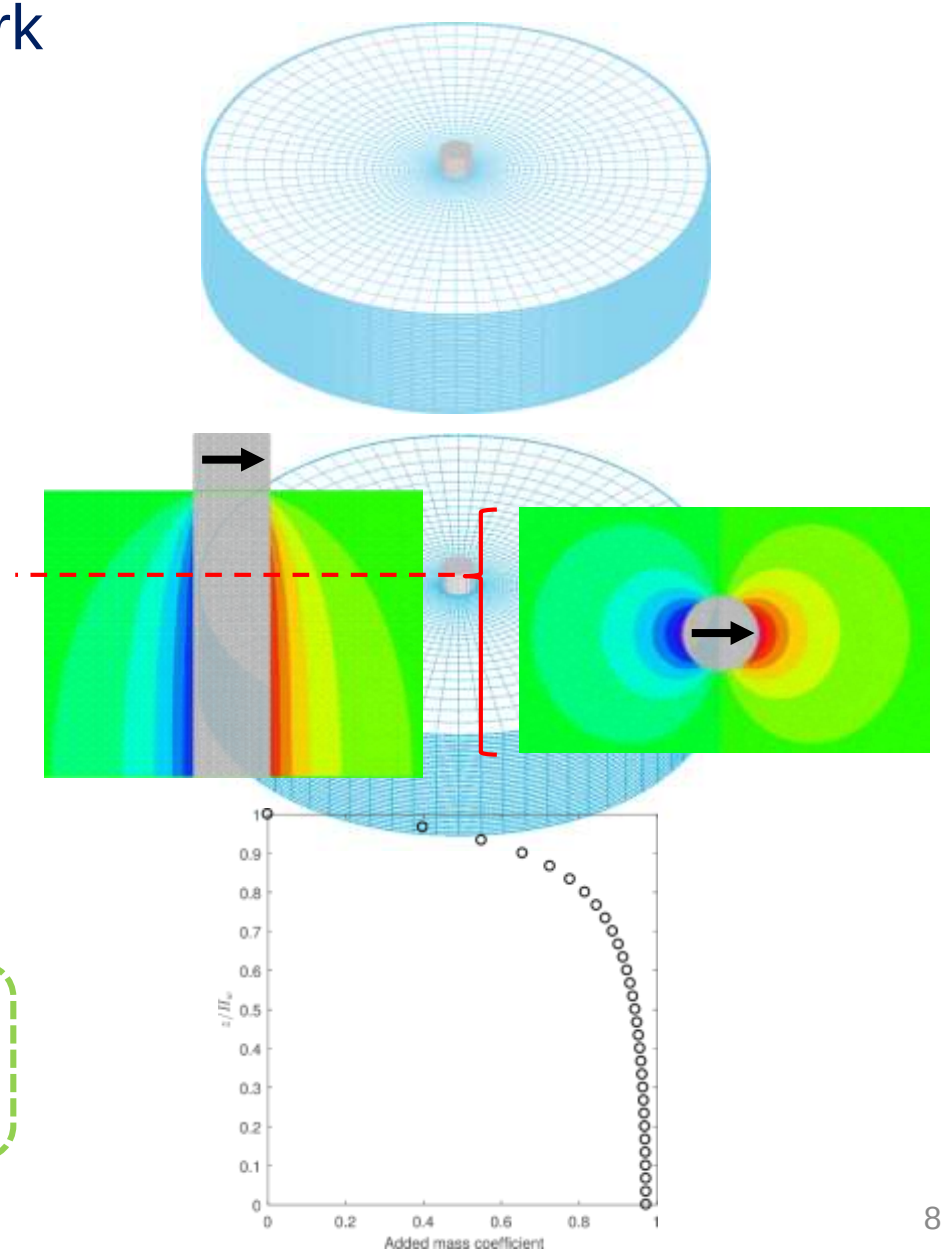
Finite element model
Column: Solid elements
Water: Potential-based fluid elements

Apply acceleration time-history to entire column



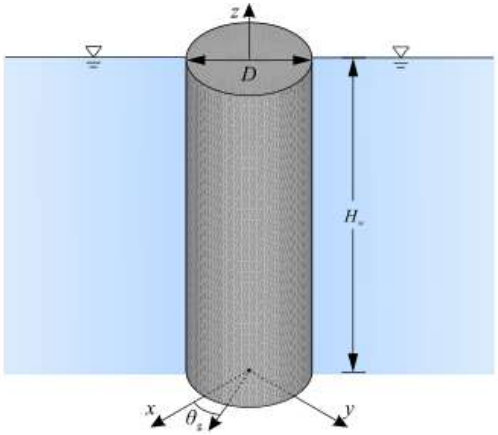
Calculate hydrodynamic pressure in water domain

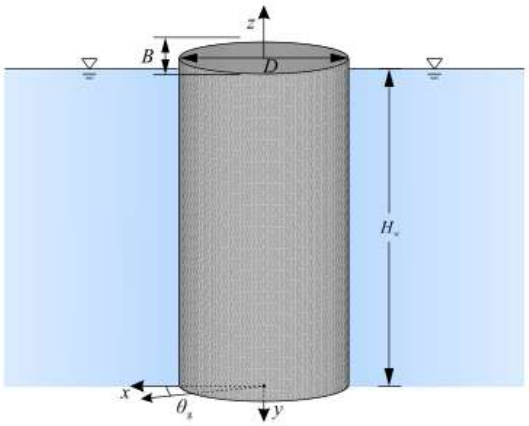
Integrate hydrodynamic pressure along column surface to calculate hydrodynamic added mass $m_a(z)$ at various pier height

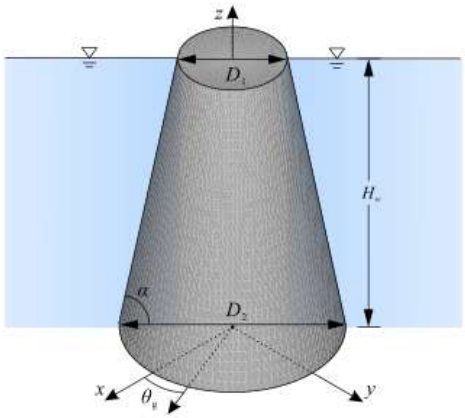


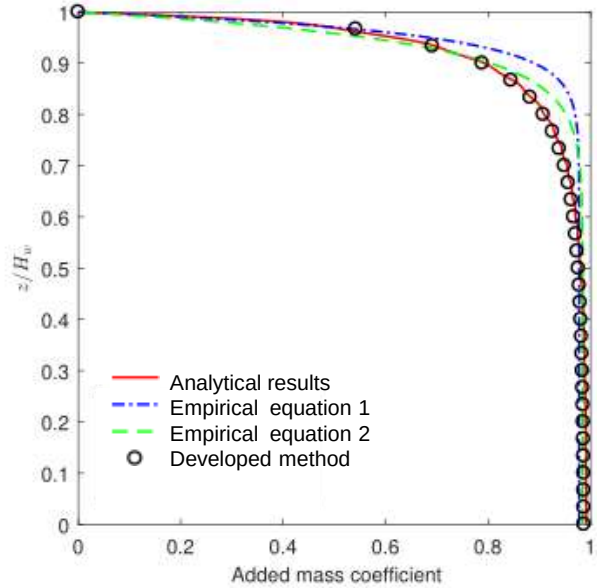
Potential-based added mass calculation

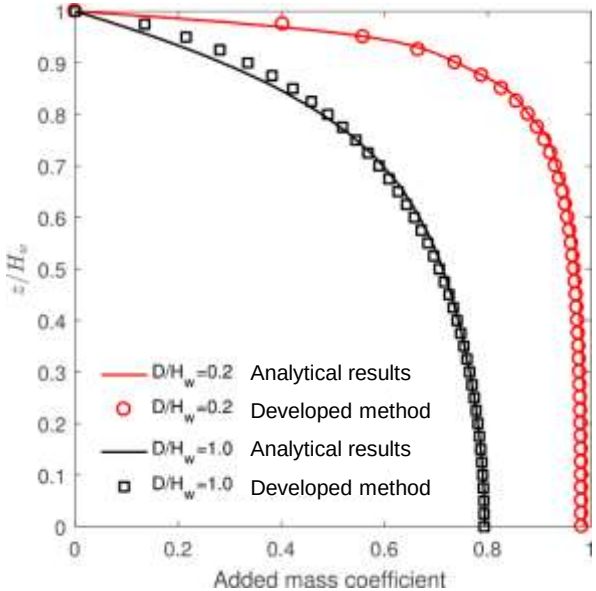
- Validation with analytical results

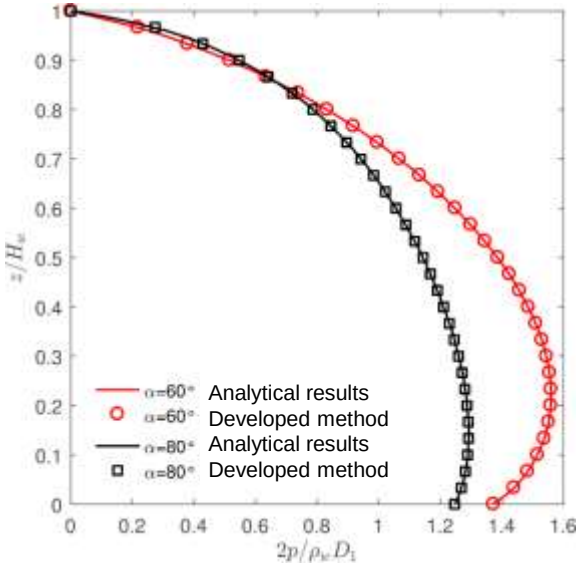








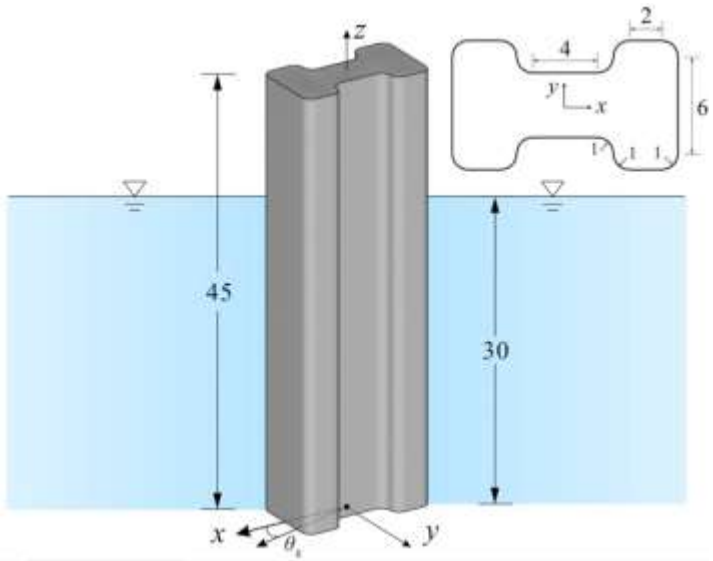




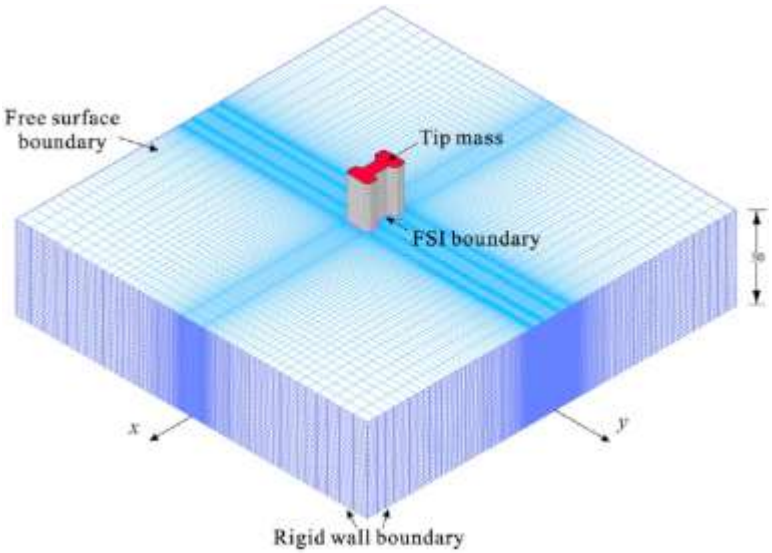
Good agreement with analytical results

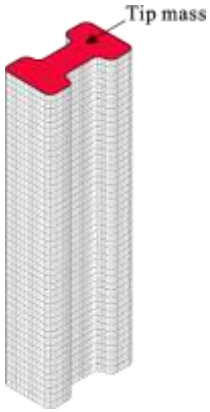
Potential-based added mass calculation

- Pier with dumbbell cross-section

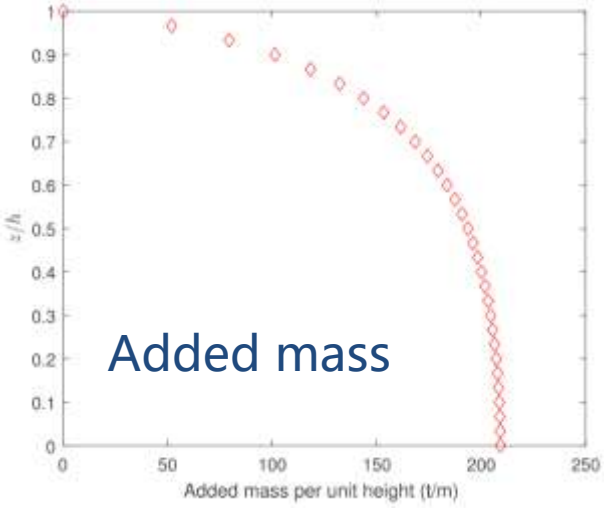


Model 1

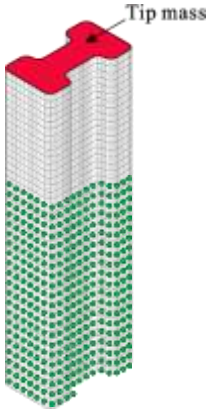




Pier



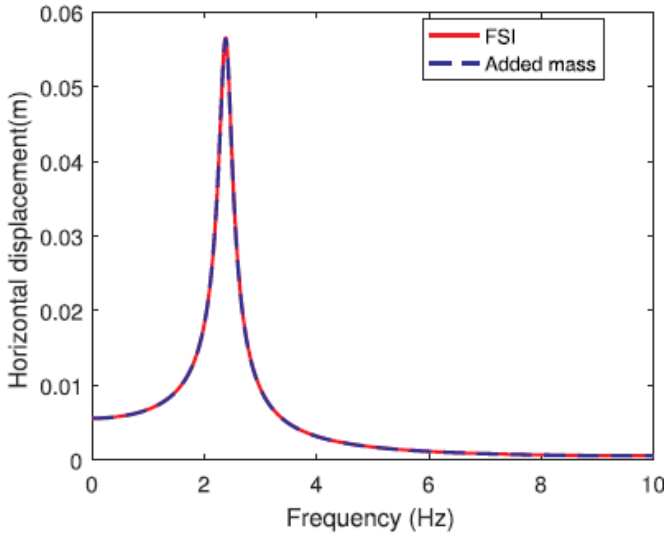
Added mass

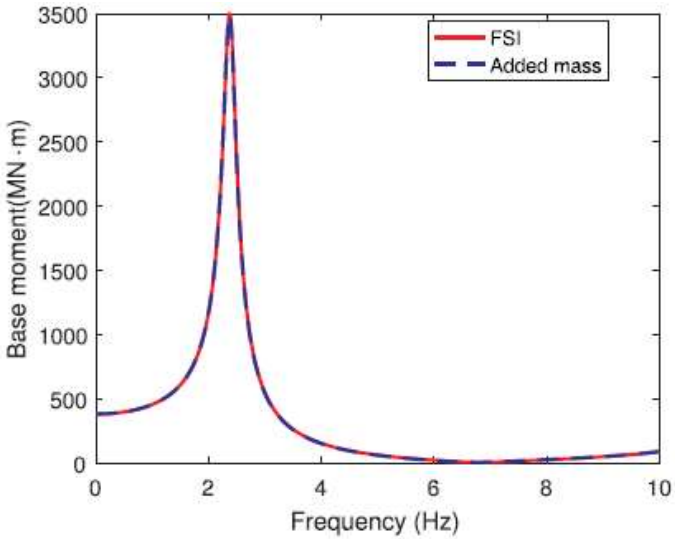


Model 2

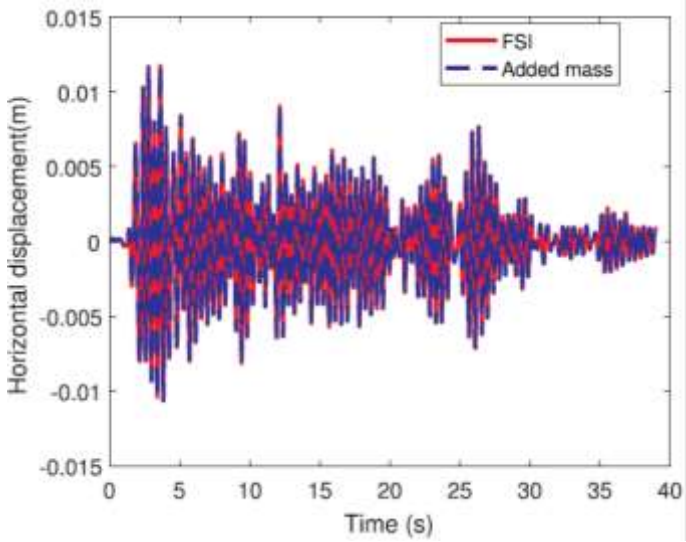
Potential-based added mass calculation

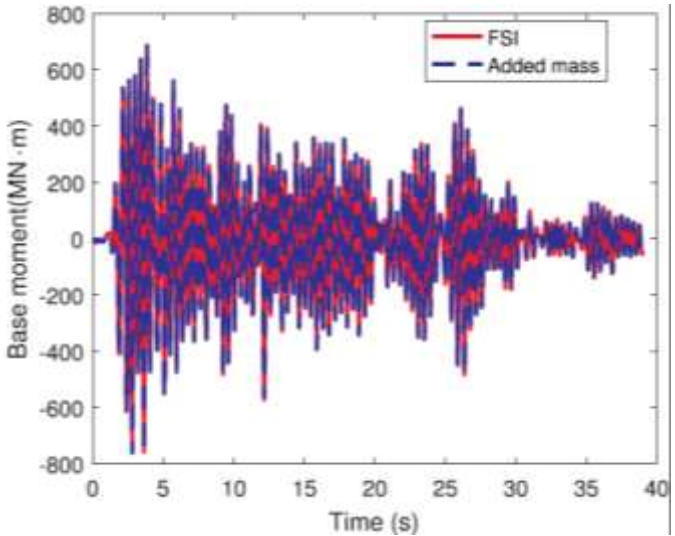
- Frequency-domain seismic responses comparison





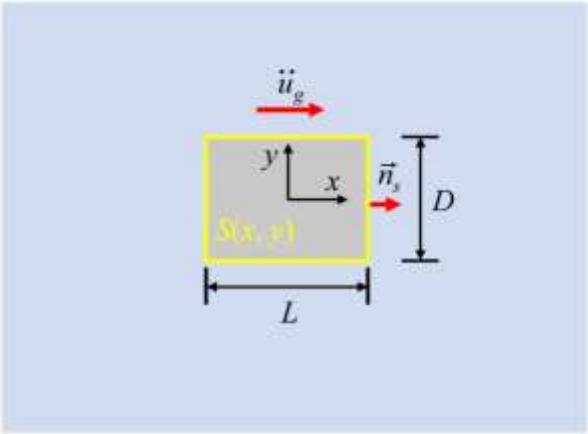
- Time-domain seismic responses comparison

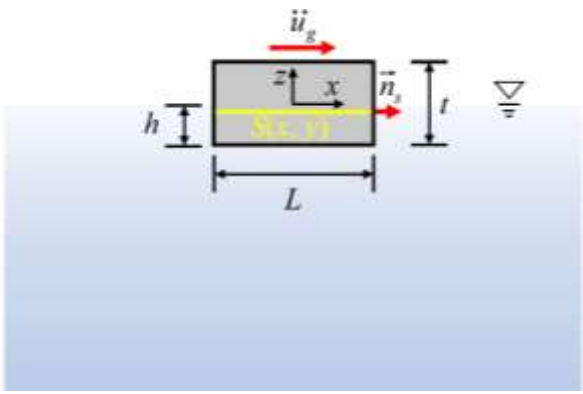




Added mass model of the cap

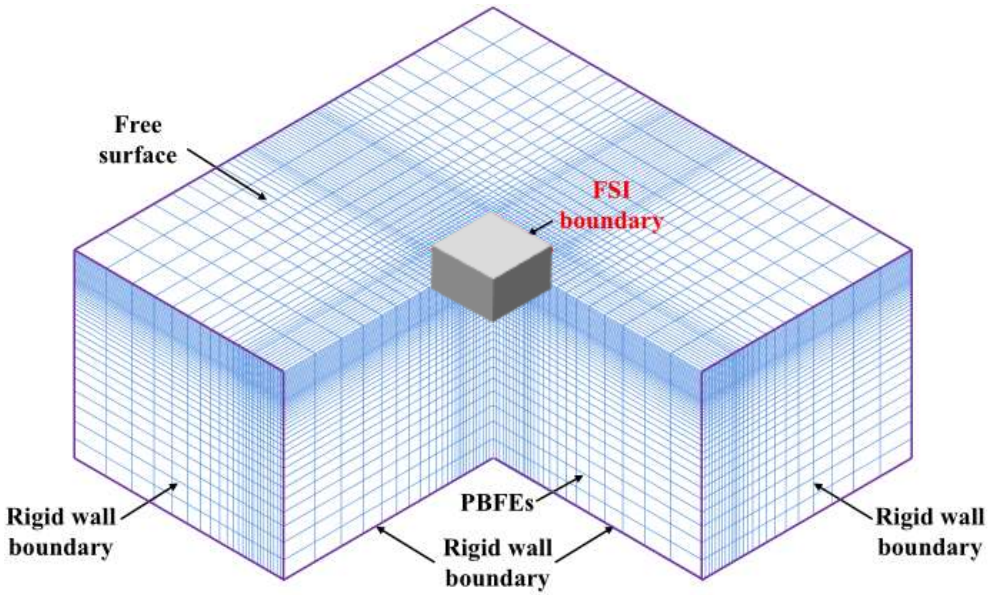
- Calculation of added mass for pile cap





Four key parameters defines a typical rectangular pile cap (D, L, t, h)

Finite element model

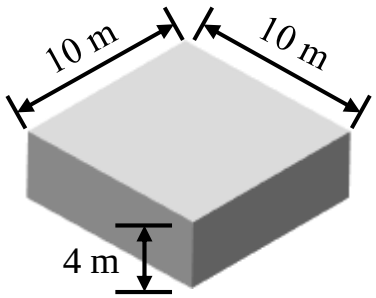


Rigid motion induced hydrodynamic pressure

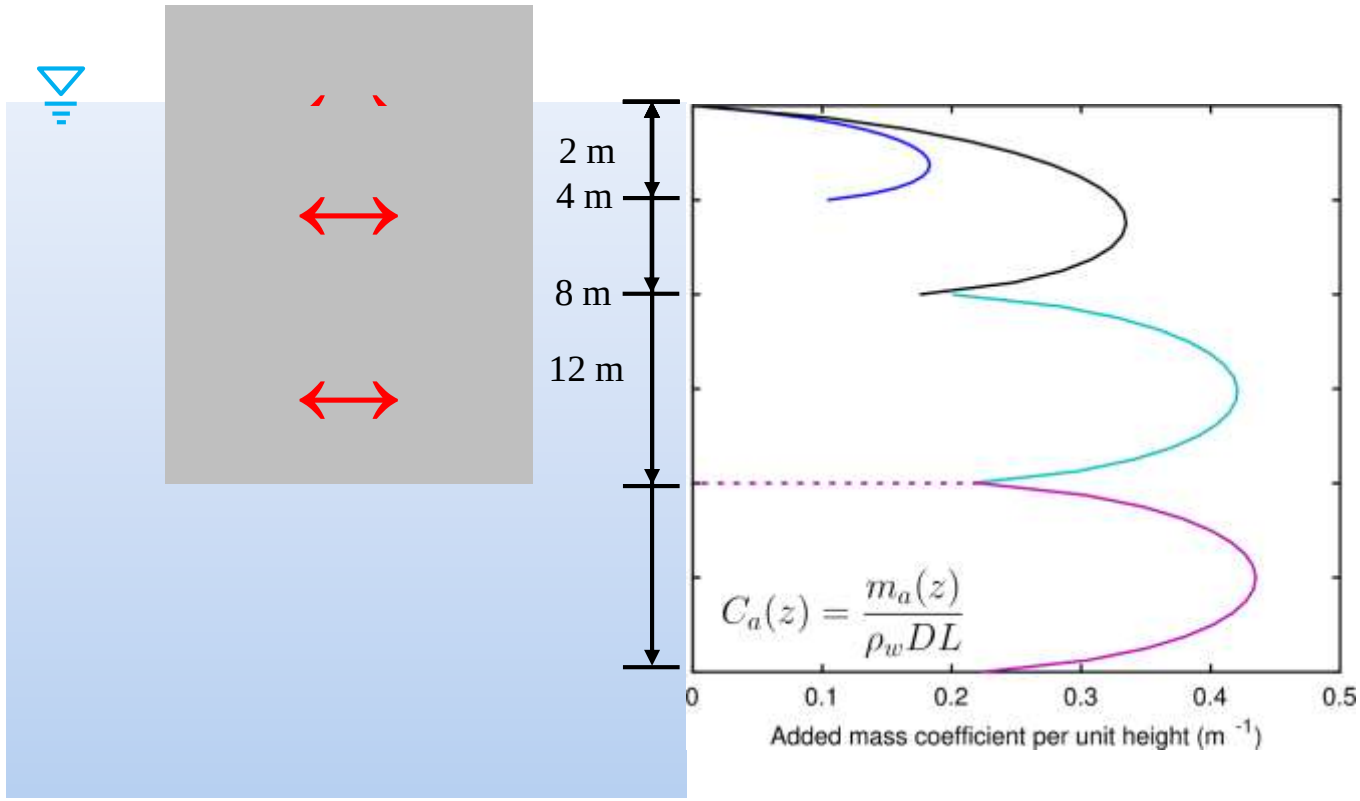
Hydrodynamic added mass $m_a(z)$

Added mass model of the cap

- Added mass coefficient of an example pile cap



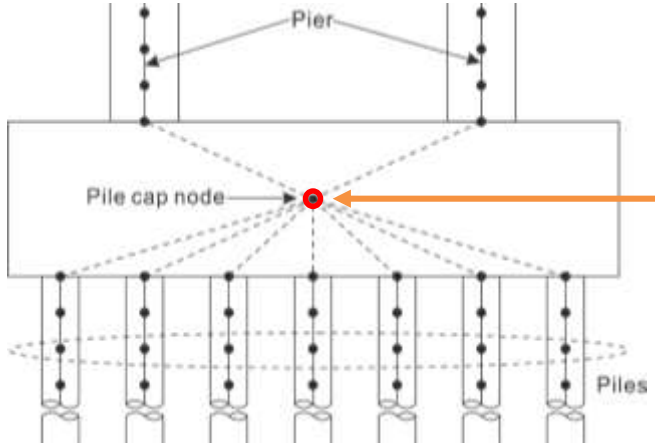
Example pile cap

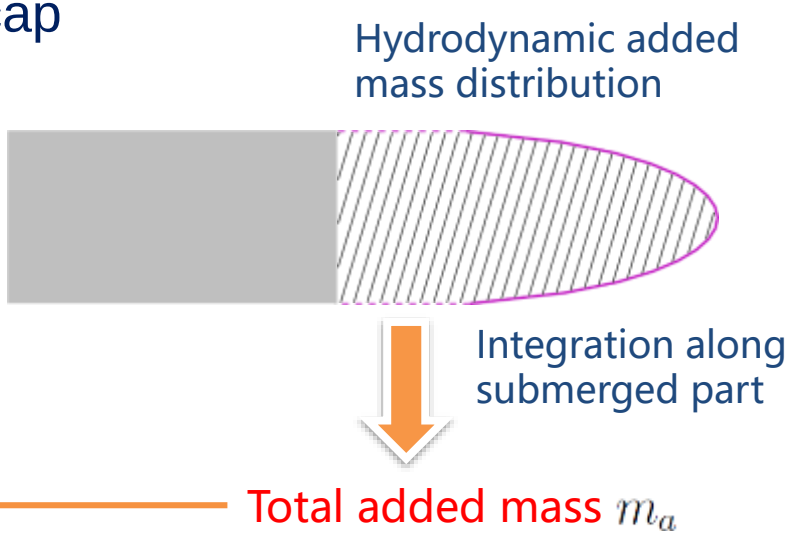


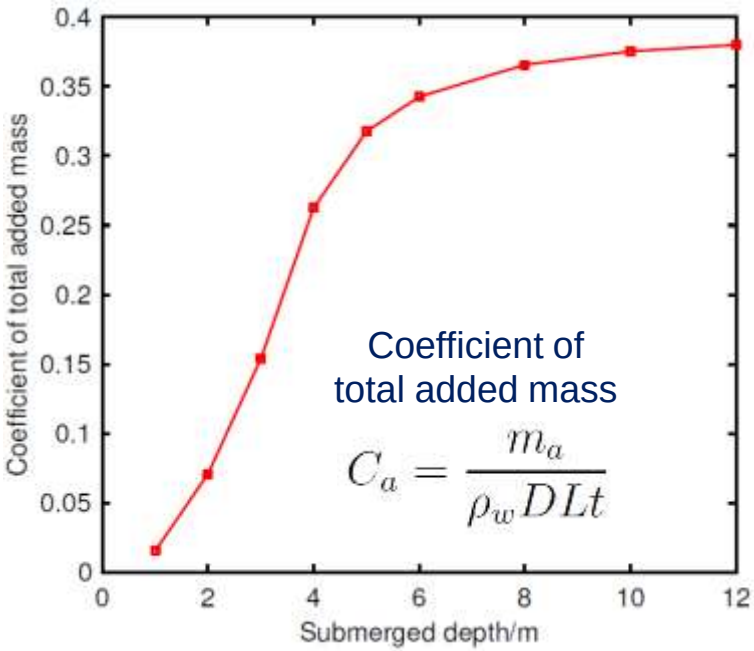
- The traditional added mass coefficient of rectangular column is larger than 1 $m_A = \boxed{C_A} \rho_w \frac{D^2}{4}$
- Added mass coefficient of rectangular pile cap is much smaller than 1
- The empirical equations will overestimate the hydrodynamic added mass of pile cap

Added mass model of the cap

- Hydrodynamic added mass of the pile cap





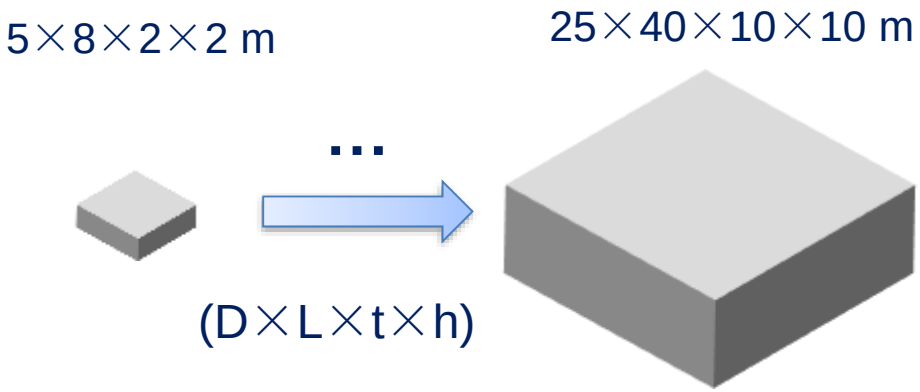


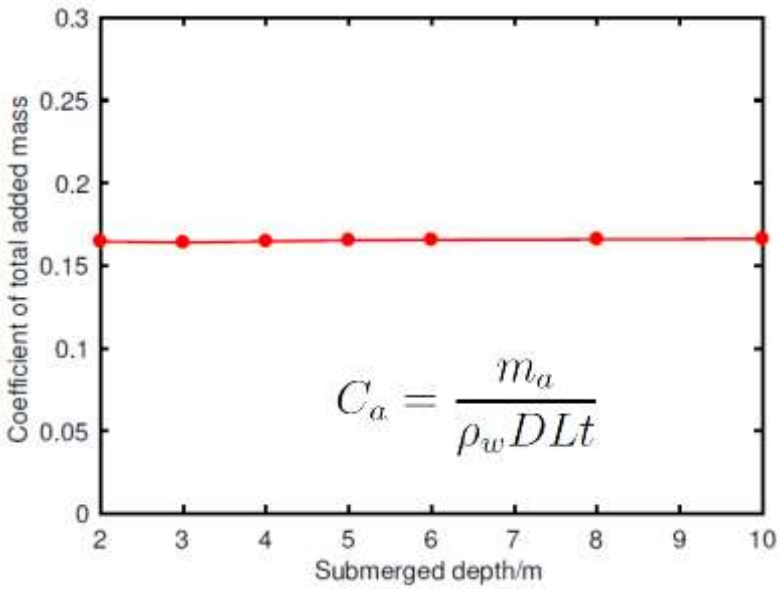
- The coefficient of total added mass firstly increases linearly with submerged depth less than 4 m
- The coefficient then remains almost unchanged with the submerged depth larger than 6 m

Added mass model of the cap

- Total added mass coefficients for a group of pile caps

A group of pile caps with same value of D/L, D/t, and h/t

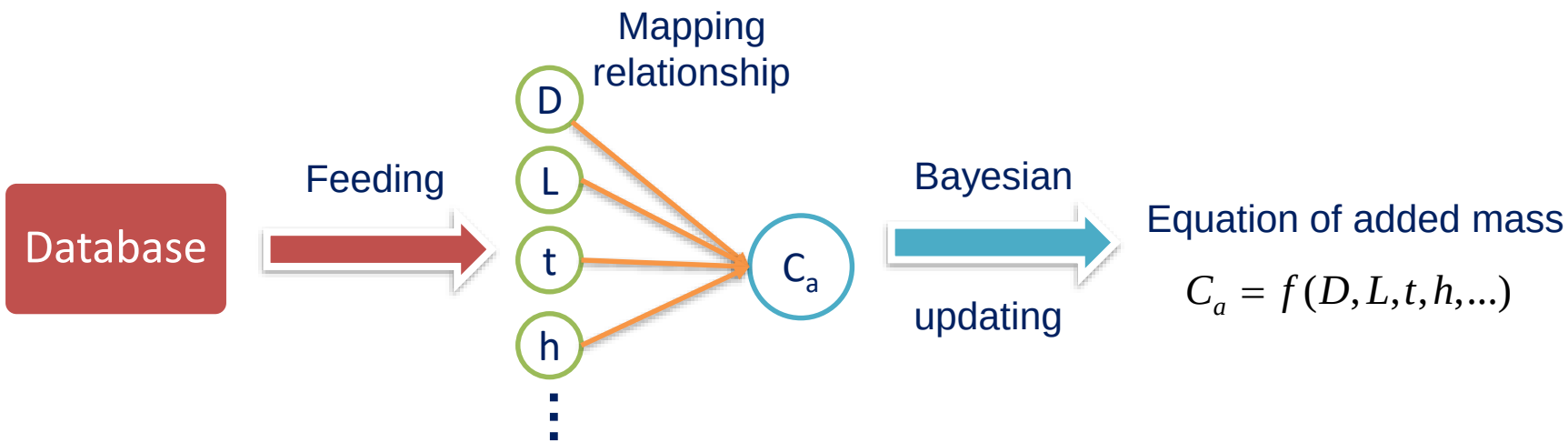




- Although the geometrical dimensions of each pile cap are different, its coefficient is identical to each other
- If the added mass of a pile cap is known, then the added mass of a group with same geometrical ratios can be computed by scaling

Added mass model of the cap

- Numerical computing involves high expertise for sophisticated FSI modelling and large computational efforts
- A practical equation is highly desired for engineering application
- Fitting becomes difficult to deal with multi-parameters
- Artificial Neural Network is a black box between the input and output
- Bayesian updating approach can provide explicit expression of mapping relationship between added mass and geometrical parameters



Added mass model of the cap

- Bayesian updating model: database generation

Parameter	Ranges	Unit
D/L	0.4-2.5	m
D/t	2-10	m
h/t	0.4-3.5	m

Cover wide ranges of pile caps in engineering practices

Latin hypercube sampling (LHS)

D = 10 m

150 rectangular pile cap samples with different D/L, D/t, h/t

Corresponding to 150 various groups of pile cap

Modeling and calculation

Database

1. Coefficients of total added mass C_a
2. Geometrical ratios (D/L, D/t, h/t)

Added mass model of the cap

- Bayesian updating model: general formulation

$$\ln[C_a(\mathbf{x}, \boldsymbol{\Theta})] = \ln[\hat{c}_a(\mathbf{x})] + \gamma(\mathbf{x}, \boldsymbol{\theta}) + \sigma\varepsilon$$

$C_a(\mathbf{x}, \boldsymbol{\Theta})$ is the added mass coefficient of rectangular pile cap

$\hat{c}_a(\mathbf{x})$ is the deterministic added mass coefficient, such as equation for pier

$\gamma(\mathbf{x}, \boldsymbol{\theta})$ is the correction term to improve the model accuracy

$\boldsymbol{\Theta} = (\boldsymbol{\theta}, \sigma)$ is a set of unknown model parameters

σ is the unknown standard deviation of the model error

Prior distribution of model parameters → Use the database to update model parameters → Posterior distribution of model parameters

Non-informative

Markov chain Monte Carlo method

Added mass model of the cap

- Bayesian updating model: Model optimization

Typical expression of correction term

$$\gamma(\mathbf{x}, \boldsymbol{\theta}) = \sum_{i=1}^{n} \theta_i k_i(\mathbf{x}) \quad k_i(\mathbf{x}), i = 1, ..., n$$

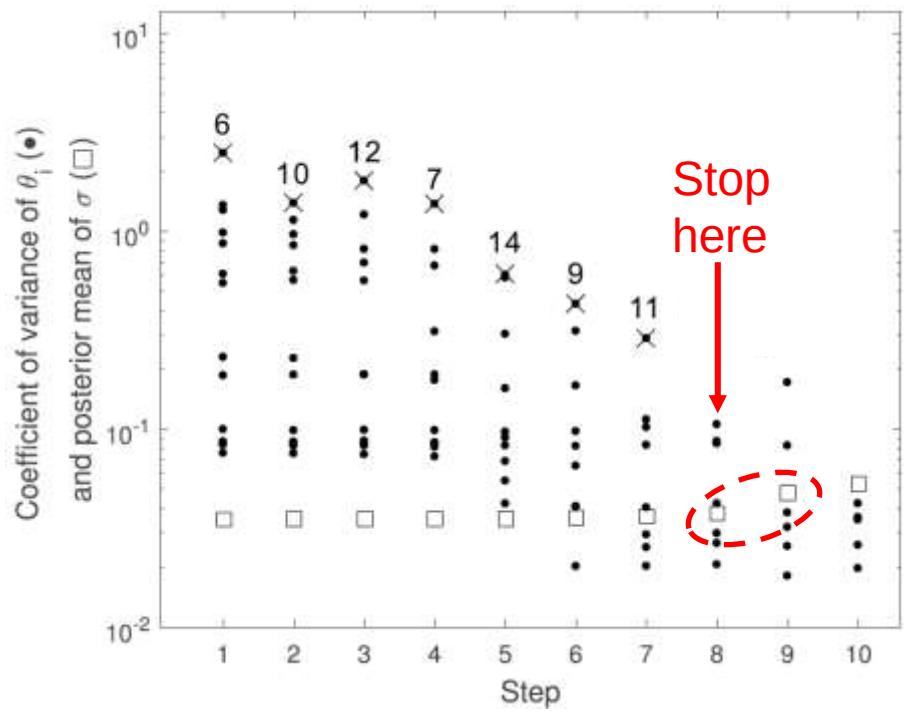
are the candidate explanatory functions to capture the inherent bias in $\hat{c}_a(\mathbf{x})$

Initial selected explanatory functions

Explanatory function	Formula
k_1	1
k_2	$\ln(D/L)$
k_3	$\ln(D/s)$
k_4	$\ln(L/t)$
k_5	$\ln(s/h)$
k_6	$[\ln(D/L)]^2$
k_7	$[\ln(D/t)]^2$
k_8	$[\ln(D/h)]^2$
k_9	$[\ln(D/s)]^2$
k_{10}	$[\ln(L/s)]^2$
k_{11}	$[\ln(L/h)]^2$
k_{12}	$[\ln(L/t)]^2$
k_{13}	$[\ln(h/t)]^2$
k_{14}	$[\ln(s/t)]^2$

$$s = \begin{cases} t & h > t \quad \text{Fully submerged} \\ h & h \leq t \quad \text{Partially submerged} \end{cases}$$

Delete unimportant terms from model step by step



Added mass model of the cap

- Bayesian updating model: final expression of model

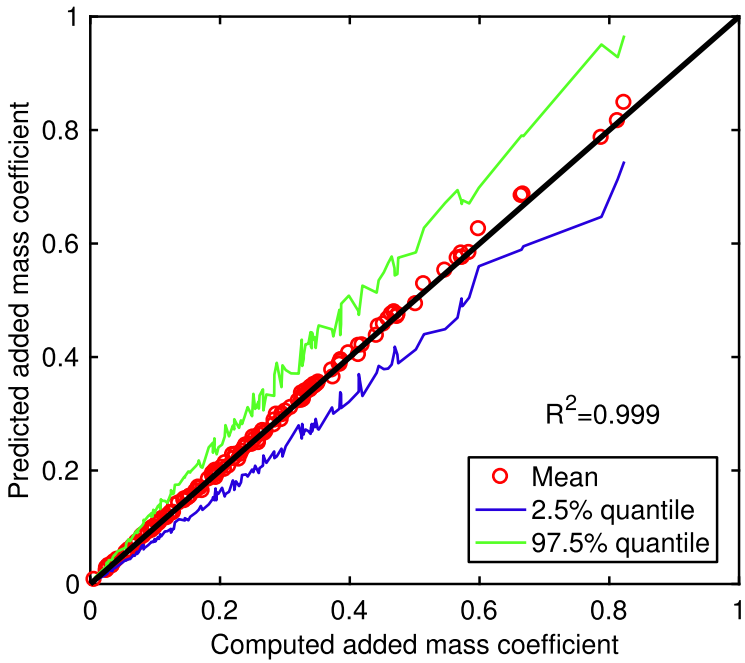
$$\ln[C_a(\mathbf{x}, \mathbf{\Theta})] = \theta_1 + \theta_2 \ln(D/L) + \theta_3 \ln(D/s) + \theta_4 \ln(L/t)$$
$$+ \theta_5 \ln(s/h) + \theta_8 [\ln(D/h)]^2 + \theta_{13} [\ln(h/t)]^2 + \sigma\varepsilon$$

- Bayesian updating model: posterior distribution

Parameter	Mean	Standard deviation	Distribution
θ_1	-0.5547	0.0164	Normal
θ_2	1.8812	0.0387	Normal
θ_3	-1.6299	0.0429	Normal
θ_4	0.9101	0.0381	Normal
θ_5	-0.5111	0.0442	Normal
θ_8	-0.077	0.0065	Normal
θ_{13}	-0.2727	0.0287	Normal
σ	-3.2822	0.05734	Log-normal

Added mass model of the cap

- Practical added mass equation for engineering application



- Most of dots distribute along the equality line
- High value of R squared
- Most of the coefficients computed using the numerical model falls in the 95 % credible intervals

$$
\begin{aligned}
m_a^{mean} &= \rho_w DLt \cdot C_a \\
&= \rho_w DLt \cdot 0.5742 \left(\frac{D}{L}\right)^{\beta_1} \left(\frac{D}{s}\right)^{\beta_2} \left(\frac{L}{t}\right)^{\beta_3} \left(\frac{s}{h}\right)^{\beta_4} \left(\frac{D}{h}\right)^{\beta_5} \left(\frac{h}{t}\right)^{\beta_6}
\end{aligned}
$$

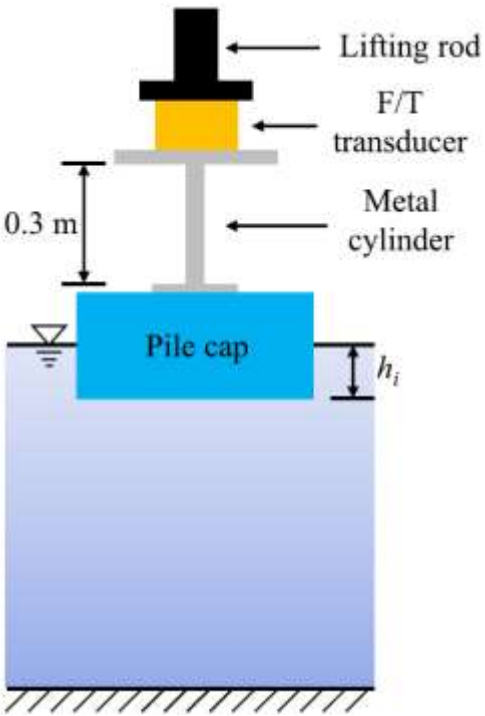
$$
\begin{cases}
\beta_1 = 1.8812 \\
\beta_2 = -1.6299 \\
\beta_3 = 0.9101 \\
\beta_4 = -0.5111 \\
\beta_5 = -0.077 \ln(D/h) \\
\beta_6 = -0.2727 \ln(h/t)
\end{cases}
$$

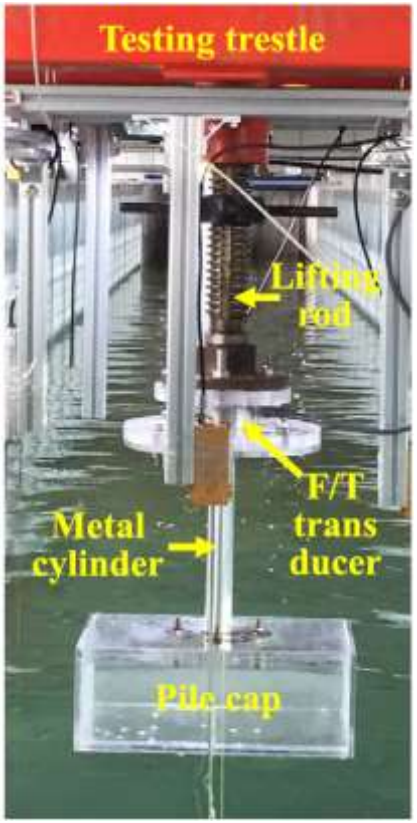
$$D/L \in [0.4, 2.5], D/t \in [2, 10], h/t \in [0.4, 3.5]$$

Application range

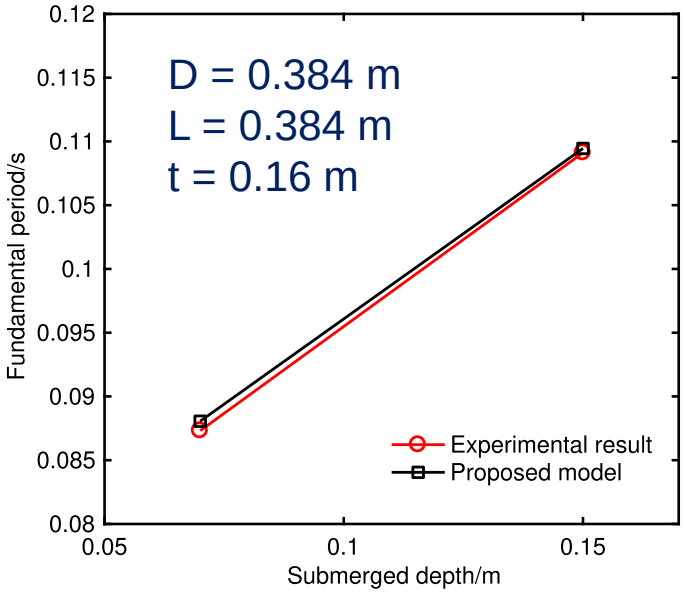
Added mass model of the cap

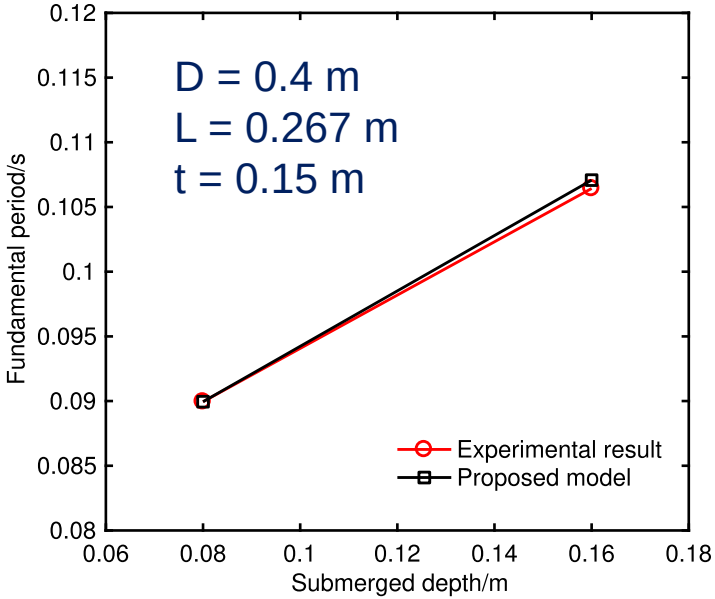
- Experimental Validation I





Free vibration test under different submerged depth in a water tank



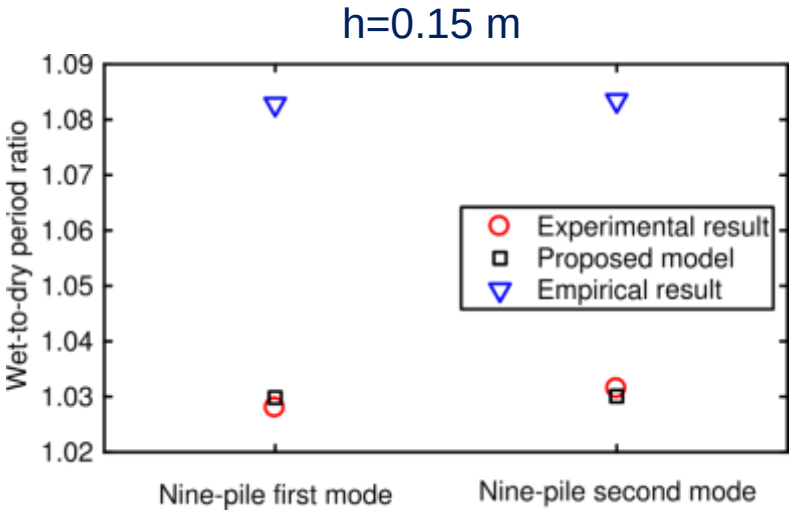


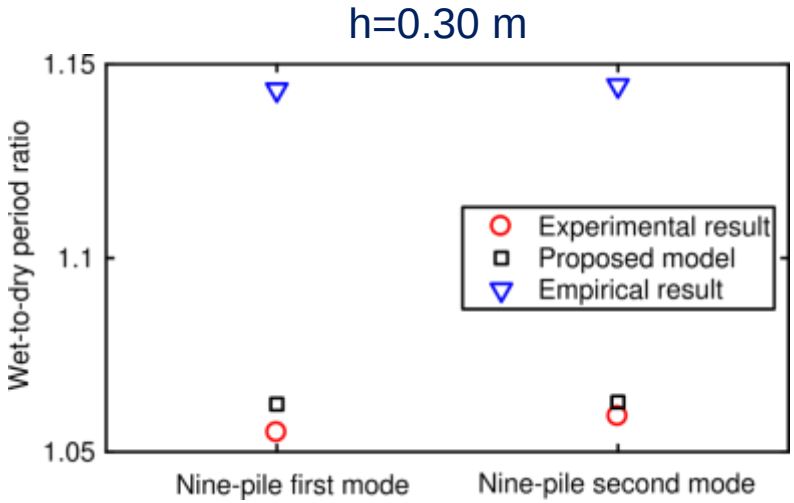
Added mass model of the cap

- Experimental Validation II

Cap: D = 0.6 m L = 0.6 m t = 0.30 m

Modal test of a reduced-scale bridge pile cap foundation in Wei et al. (2013)





Kai Wei, Wancheng Yuan, and Najib Bouaanani. "Experimental and numerical assessment of the three-dimensional modal dynamic response of bridge pile foundations submerged in water." *Journal of Bridge Engineering* 2013.

Concluding Remarks

Summary

- ✓ A general method to determine the hydrodynamic added mass was developed
- ✓ A Bayesian updating model for added mass of rectangular pile cap was proposed
- ✓ Experimental modal tests of pile cap specimens were used to validate the model

Future work

- ☐ Models for pile cap with more complex cross-section

International Association Bridge Earthquake Engineering

西南交通大學 Southwest Jiaotong University

Thanks for attention!

Acknowledgements

Collaborators: Jiarui Zhang, Prof. Shunquan Qin

Financial support from Natural science foundation of China (51708455 and 51978578)

Email: kaiwei@swjtu.edu.cn